THÈSE

Pour obtenir le grade de

DOCTEUR DE L'UNIVERSITÉ GRENOBLE ALPES

École doctorale : MSTII - Mathématiques, Sciences et technologies de l'information, Informatique
Spécialité : Informatique
Unité de recherche : Laboratoire des Sciences pour la Conception, l'Optimisation et la Production de Grenoble

Algorithmes pour des Problèmes de Coloration avec Promesse sur les Tournois et les Graphes

Algorithms for Promise Coloring Problems on Tournaments and Graphs

Présentée par :

Felix KLINGELHOEFER

Direction de thèse :

Louis ESPERET — Directeur de thèse
DIRECTEUR DE RECHERCHE, CNRS DELEGATION ALPES

Alantha NEWMAN — Co-encadrante de thèse
CHARGEE DE RECHERCHE, CNRS DELEGATION ALPES

Rapporteurs :

PIERRE CHARBIT
MAITRE DE CONFERENCES HDR, UNIVERSITE PARIS CITE

SANJEEV KHANNA
PROFESSEUR, UNIVERSITY OF PENNSYLVANIA

Thèse soutenue publiquement le **4 décembre 2023**, devant le jury composé de :

PIERRE CHARBIT — Rapporteur
MAITRE DE CONFERENCES HDR, UNIVERSITE PARIS CITE

SANJEEV KHANNA — Rapporteur
PROFESSEUR, UNIVERSITY OF PENNSYLVANIA

CARLA GROENLAND — Examinatrice
ASSISTANT PROFESSOR, DELFT UNIVERSITY OF TECHNOLOGY

ZOLTAN SZIGETI — Président
PROFESSEUR DES UNIVERSITES, GRENOBLE INP

STEPHAN THOMASSE — Examinateur
PROFESSEUR DES UNIVERSITES, ENS DE LYON

Invités :

LOUIS ESPERET
DIRECTEUR DE RECHERCHE, CNRS DELEGATION ALPES

ALANTHA NEWMAN
CHARGEE DE RECHERCHE, CNRS DELEGATION ALPES

Abstract

The first part of this thesis is on the subject of coloring tournaments, from an algorithmic, complexity and structural perspective. A *k-coloring* of a directed graph, and in particular a tournament, is a partition of its vertices into k acyclic sets. The chromatic number of a directed graph or a tournament is then the minimum k such that it is k-colorable. Deciding if a tournament is 2-colorable is already NP-hard. A natural problem, akin to that of coloring a 3-colorable graph with few colors, is to color a 2-colorable tournament with few colors. This problem does not seem to have been addressed before, although it is a special case of coloring a 2-colorable 3-uniform hypergraph with few colors, which is a well-studied problem with super-constant lower bounds.

We present an efficient decomposition lemma for tournaments and show that it can be used to design polynomial-time algorithms to color various classes of tournaments with few colors, including an algorithm to color a 2-colorable tournament with ten colors. For the classes of tournaments considered, we complement our upper bounds with strengthened lower bounds, painting a comprehensive picture of the algorithmic and complexity aspects of coloring tournaments. We then extend our results to different classes of tournaments and digraphs.

The neighborhood of an arc uv in a tournament T is the set of vertices that form a directed triangle with arc uv. By using our decomposition lemma, we show that if the neighborhood of every arc in a tournament has bounded chromatic number, then the whole tournament has bounded chromatic number. This holds more generally for oriented graphs with bounded independence number, and we extend our proof from tournaments to this class of dense digraphs. As an application, we prove the equivalence of a conjecture of El-Zahar and Erdős and a recent conjecture of Nguyen, Scott and Seymour relating the structure of graphs and tournaments with high chromatic number.

In the second part of this thesis, we focus on the problem of finding maximum stable sets in the class of CYCLE-PLUS-TRIANGLES graphs. A CYCLE-PLUS-TRIANGLES graph is the disjoint union of t triangles and a Hamilton cycle on the $3t$ vertices. It is 3-colorable, and we give an overview of the different proofs of its 3-colorability. There is, however, no known efficient algorithm to find a 3-coloring or even to find a maximum stable set (i.e., a stable set of size t).